**Conjugate Heat Transfer in Air-to-Refrigerant Airfoil Heat Exchangers**

A light and compact heat exchange system was realized using two air-to-refrigerant airfoil heat exchangers and a recirculated heat transport refrigerant. Its heat transfer performance was experimentally investigated. Carbon dioxide or water was used as a refrigerant up to a pressure of 30 MPa. Heat transfer coefficients on the outer air-contact and inner refrigerant-contact surfaces were calculated using an inverse heat transfer method. Correlations were developed for the Nusselt numbers of carbon dioxide and water on the inner refrigerant-contact surface. Furthermore, we proposed a method to evaluate a correction factor corresponding to the thermal resistance of the airfoil heat exchanger.

**Keywords:** Nusselt number, supercritical carbon dioxide, gas turbine, airfoil heat exchanger, intercooler, recuperator

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**Introduction**

Intercooled, recuperated aviation gas turbines (IR gas turbines) have the potential to reduce fuel consumption in aviation. Intercoolers and recuperators enhance the gas turbine cycle efficiency from a thermodynamic point of view. Wilfert et al. constructed a medium pressure gas turbine components demonstrator that had the potential to achieve a 17% reduction in fuel consumption compared with a baseline gas turbine. However, they noted that adapting recuperators, in particular, for a practical aviation gas turbine remained a future challenge [1]. Conventional IR gas turbines are too heavy for use as aviation gas turbines because they use a type of tube matrix air-to-air heat exchanger [2] or a type of primary surface air-to-air heat exchanger [3]. Although both types have high temperature effectiveness, they are heavy. Furthermore, hot and cold air must be collected at the heat exchanger. Therefore, long and heavy air connecting ducts are required.

To overcome these problems, a heat exchange system using a heat transport liquid or supercritical refrigerant (abbreviated in this paper to “refrigerant”) may be used, as shown in Fig. 1. In this system, the refrigerant transports heat from the hot section to the cold section. Therefore, the hot and cold sections can be installed at separate sites. In the intercooling system, heat exchanger A works as an air cooler, and heat exchanger B works as a radiator. Conversely, in the recuperating system, heat exchanger A works as a heat absorber, and heat exchanger B works as an air heater. In addition, heat transport liquid and supercritical refrigerants have higher densities and greater specific heat values than air. Therefore, the refrigerant connecting tubes require a much smaller diameter than air connecting ducts. Although an additional recirculation pump is needed, it must only drive the refrigerant against the pressure loss of the refrigerant loop. Ito proposed an IR aviation gas turbine that uses this heat exchange system in the form of heat exchangers installed in already equipped components in a baseline aviation gas turbine, to reduce its weight [4]. In the intercooling system of this design, fixed stators and vanes in the compressor are used as the air cooler, and vanes in the bypass duct are used as the radiator. In the recuperating system, vanes in the combustor are used as the air heater, and vanes in the core nozzle are used as the heat absorber. Here, the working airflow path can remain in the same position as a baseline aviation gas turbine. Therefore, there is no additional pressure loss in the working airflow. Because vanes are used as heat exchangers, this type of heat exchanger is hereafter referred to as an “airfoil heat exchanger.”

An airfoil heat exchanger is physically similar to the vanes of conventional air-cooled HPT vanes [5]. Therefore, the heat transfer performance of an airfoil heat exchanger will be discussed compared with that of air-cooled HPT vanes. However, it is difficult to evaluate the heat transfer performance of real heat exchanger components. Bejan developed the constructal theory of design for cooling fins [6], and Lorenzini and his colleague energetically applied this theory to optimize Y-shaped and I-shaped fins [7]. In addition, they conducted computer fluid dynamics calculations of the heat transfer for arrays of Y-shaped, I-shaped, and T-shaped fins, and optimized the results [8]. Moreover, the optimized results were compared with the constructal theory’s results [9,10]. Similarly, modern compressor stators and guide vanes are three-dimensional airfoils optimized by computer fluid dynamics; however, a three-dimensional airfoil is too complex to use in experiments. Therefore, we chose a two-dimensional NACA65-(12A218b)10 airfoil. An NACA65 series airfoil is a traditional two-dimensional airfoil for a compressor stator. The estimated heat transfer rates are more suitable than those of a simpler plane for realizing the airfoil heat exchanger. We prepared a cascade of three NACA65-(12A218b)10 airfoils, each of which had five inner refrigerant channels, as shown in Fig. 2, as a test model of the airfoil heat exchanger. This cascade was installed in a subsonic wind tunnel at Mach 0.55–0.62. On the airfoil surfaces, as described by Nishiyama [11], a developing boundary layer changes from a laminar boundary layer to a turbulent boundary layer across the minimum pressure point X_{min}, i.e., the maximum point of pressure coefficient S. This is because the boundary layer is stable in regions with favorable pressure gradients but unstable in those with adverse pressure gradients.

To obtain the heat transfer coefficients on the outer and inner surfaces, Turner employed a heat conduction numerical analysis using 31 discrete surface temperatures measured in experiments [12]. In contrast, we used an inverse heat transfer method. It was conducted under conditions that included the pressure distribution around an airfoil already reported by Dunavant et al. [13] and our experimentally measured temperatures at four points. This was easier than Turner’s method because it is difficult to accurately
measure surface temperatures without disturbing a fast airflow. Furthermore, Turner used air as the inner cooling fluid, whereas we used a refrigerant. Lorenzini and Moretti pointed out that a liquid always performs better than air if the focus is exclusively on heat removal maximization [14]. Their figure seemed that there was a greater change in the temperature distribution in the fins when using a liquid than when using air. This difference may affect the heat transfer performance of an airfoil heat exchanger.

An additional contribution of our study involves the use of either supercritical carbon dioxide or compressed water as the inner cooling fluid for an airfoil heat exchanger, rather than the use of air. Liao and Zhao experimentally investigated the heat transfer coefficient of supercritical carbon dioxide in the range of 7.4–12 MPa [15]. Our study extended Liao and Zhao’s pressure range up to 30 MPa for carbon dioxide, and we also considered compressed (but not supercritical) water at pressures up to 30 MPa.

It is expected that this will help to clarify a more suitable method for estimating the heat transfer coefficients when designing airfoil heat exchangers.

**Experimental Setup**

To experimentally estimate the heat transfer performance in a high-speed compressible flow, the Reynolds, Mach, and Prandtl numbers should match those of the real gas and refrigerant flows. If scale-model experiments are conducted, all of the viscosity, thermal conductivity, specific heat, and sound speed ratios of the tested fluids should be the same as those of the real gas and refrigerant. However, rather than using such fluids in scale-model experiments, it is easier to prepare a real gas, real refrigerant, and real-size airfoil heat exchanger under real conditions.

**Wind Tunnel.** A closed-return wind tunnel at the Tokyo Institute of Technology was employed to produce a subsonic airflow. A BE-H125 Roots blower (made by ANLET Co. Ltd.) was used as the continuous air source. The nozzle outlet size was 60 × 30 mm. Its Mach number capabilities ranged from 0 to 0.8, and Mach numbers of 0.55–0.62 were chosen for testing the airfoil heat exchanger in an aviation gas turbine. The inlet airflow was sufficiently turbulent because the inlet Reynolds number Re_nozzle, whose representative length was the hydraulic diameter of the nozzle outlet, ranged from 3.92 × 10^5 to 4.52 × 10^5. In addition, the roots blower generated a turbulent airflow. Although this airflow condition involved no wakes from the preceding airfoils, unlike in a practical axial gas turbine, it was sufficient for the time-averaged airflow condition to be applied to the design of airfoil heat exchangers.

**Airfoil Heat Exchangers.** NACA 65-(12A10k)10 airfoils with inner refrigerant channels were employed as test models, as shown in Fig. 2. The airfoils were made of SUS304 because its thermal conductivity is 16 W/K·m, which is almost the same as the thermal conductivity of the materials used for practical compressors or vanes; for example, approximately 20 W/K·m for titanium alloy and 11–21 W/K·m for nickel-based heat-resistant alloy. The chord length was 44 mm, and the width was 28 mm. These are average sizes for the stators or vanes in the compressor section of middle or large class aviation gas turbines.

Four type-K thermocouples with a diameter of 0.5 mm were installed in four taps with a 0.7 mm inner diameter and used to measure the temperature distribution of each airfoil heat exchanger. All were located at midspan. As seen in Fig. 2, the airfoil was deployed at an incidence ζ = 0 (i.e., a flow direction angle from the airfoil camber line at its leading edge, corresponding to an angle-of-attack ψ = 9.47 deg, i.e., an inlet flow direction angle from the airfoil chord). x and y axes were defined in the horizontal and vertical directions. Thermocouples T_1 and T_2 were located on the camber line at x = 3 and 41 mm, respectively. Thermocouples T_3 and T_4 were located 1.2 mm below and above the camber line at x = 22 mm, respectively. Here, to enhance the temperature measurement accuracy and detect temperature differences smaller than 1 K among T_i-T_{i-1}, the potential differences between them were measured directly. This measurement method enhances the accuracy when detecting small temperature differences. To calibrate all thermocouples, considering the digital voltage meter error, all of the thermocouples’ tips were placed in ice water at a constant temperature of 273.15 K. We developed data acquisition PC software to cancel out temperature drifts from 273.15 K. Therefore, an accuracy of ±0.025 K was achieved for the temperature differences among all of the thermocouples.

**Test Section Configurations for Cascade of Airfoil Heat Exchangers.** Figure 3 shows the configurations of a cascade of three NACA65-(12A10k)10 airfoil heat exchangers. Three airfoils were deployed at the same positions as some of those tested by...
The refrigerant flow loop is shown in Fig. 4. The refrigerant was pumped up into the refrigerant flow loop via an 8800 series plunger pump (made by L. TEX Corporation). The refrigerant was supercritical carbon dioxide or compressed liquid water could be used as the refrigerant. The refrigerant was pumped up into the refrigerant flow loop via an 8800 series plunger pump (made by L. TEX Corporation).

**Inverse Heat Transfer Method**

The experimentally obtained data alone were not sufficient to enable us to determine the air and refrigerant heat transfer coefficients \( h_{air} \) and \( h_{ref} \). This was because the surface temperature distributions on the outer air-contact surfaces and inner refrigerant-contact surfaces should be known to accurately evaluate \( h_{air} \) and \( h_{ref} \). However, these were not measured. Therefore, in order to find the best combination of \( h_{air} \) and \( h_{ref} \), an inverse heat transfer method and a least square method were used to explain the experimentally obtained data.

**Airfoil Heat Exchanger Temperature.** There were only four thermocouples in the airfoil heat exchanger, and these were not located on the surfaces. Therefore, the inverse heat transfer method was applied to estimate \( h_{air} \) and \( h_{ref} \). Figure 5 shows the two-dimensional control volumes of the airfoil heat exchanger used to perform the inverse heat transfer method by a numerical analysis of the heat conductance in the airfoil heat exchanger. For each control volume \( j \), the finite control volume method was employed. Namely, the steady state basic equation in an integrated form is

\[
Q_{\text{cond,air}} = Q_{\text{conv,air}} + Q_{\text{air}}
\]

Here, the left-hand term means the heat conductance rate of the solid part of the airfoil heat exchanger. The right-hand terms

\[
\begin{align*}
T_{w} & = \frac{1}{\frac{1}{T_{a}} + \frac{1}{T_{i}}} \\
T_{i} & = \frac{1}{\frac{1}{T_{a}} + \frac{1}{T_{o}}} \\
T_{o} & = \frac{1}{\frac{1}{T_{a}} + \frac{1}{T_{i}}}
\end{align*}
\]
indicate the heat transfer rates through the inner refrigerant-contact surfaces (along the five bigger circular regions in Fig. 5) and outer air-contact surfaces (along the peripheral region in Fig. 5). As an example, we focus on control volume j whose neighbor control volumes are p, q, r, and s. The discretized heat conduction rate \( Q_{\text{conduction}, j} \) is

\[
Q_{\text{conduction}, j} = Q_{\text{conduction}, j-p} + Q_{\text{conduction}, j-q} + Q_{\text{conduction}, j-r} + Q_{\text{conduction}, j-s}
\]  

(2)

of course, the number of neighbor control volumes can be any number instead of four. For example, the heat conduction rate \( Q_{\text{conduction}, j-p} \) between control volume j and control volume p is

\[
Q_{\text{conduction}, j-p} = -A_{j-p}k_{\text{solid}}\frac{dT_{\text{solid}}}{dz} \approx -A_{j-p}k_{\text{solid}} \frac{T(p) - T(j)}{d_j-p}
\]  

(3)

where \( k_{\text{solid}} \) is the thermal conductivity of the solid material of the airfoil heat exchanger, \( z \) is a local coordinate along the line that goes through the centers of control volumes j and p, \( d_{j-p} \) is the distance between the centers of control volumes j and p, and \( A_{j-p} \) is the projection area of the interfacial surface area on a plane normal to the \( z \) axis. In the case of the control volume next to a refrigerant or air boundary, the right-hand terms in Eq. (1) have values; otherwise they are zero. When control volume j contacts the refrigerant, for example, the discretized heat transfer rate is as follows:

\[
Q_{\text{ref}, j} = A_{\text{ref}, j}h_{\text{ref}, j} \Delta T_{\text{ref}, j} = A_{\text{ref}, j}h_{\text{ref}, j} \left[ T_{\text{ref}, j} - T(j) \right]
\]  

(4)

where \( A_{\text{ref}, j} \) is the contact surface area between control volume j and the refrigerant, \( h_{\text{ref}, j} \) is the local heat transfer coefficient, and \( T_{\text{ref}, j} \) is the refrigerant temperature of the \( n \)th section in contact with control volume j. Here, \( n \) is any location from E to I in Fig. 4. Similar procedures

\[
Q_{\text{air}, j} = A_{\text{air}, j}h_{\text{air}, j} \Delta T_{\text{air}, j} = A_{\text{air}, j}h_{\text{air}, j} \left[ T_{\text{air}, j} - T(j) \right]
\]  

(5)

should be applied for control volume j in contact with air, where \( T_{\text{air}, j} \) is the adjacent local adiabatic air temperatures in contact with control volume j. It depends on the air boundary layer condition, namely, whether this is laminar or turbulent.

**Refrigerant Temperature.** The refrigerant properties were calculated using the procedures reported in Refs. [18,19] for carbon dioxide and [20] for water. The properties had accuracies of \( \pm 2\% \) across the critical point. In the other regions, the accuracies were better. Figure 4 shows the refrigerant tubes’ length \( L_{DE} \) and inner diameter \( D_{ref} \). The refrigerant flow is usually turbulent through the airfoil heat exchanger in new IR aviation gas turbines because a large heat flow rate per unit flow rate is preferable to a small pressure loss. Here, as shown in Fig. 4, each position is numbered to facilitate the discussion as follows: A, inlet pressure sensor; B, cross fitting; C, inlet thermocouple; D, beginning point of flow rate meter by differential pressure; E, its ending point just upstream of the central airfoil heat exchanger; F, ending point of the first u-bend section; G, of the second; H, of the third; I, that of the fourth; and J, outlet thermocouple. Section DE was the flow rate meter, which was made of a smooth tube. The refrigerant pressure loss \( \Delta P_{\text{loss}, DE} \) was measured. The refrigerant velocity \( u_{\text{ref}, DE} \) was calculated using the Darcy–Weisbach equation and Blasius’ friction coefficient in a smooth tube for turbulent conditions as follows:

\[
u_{\text{ref}, DE} = \left[ \frac{D_{\text{ref}, DE}^{1.25}}{0.1582 \rho_{\text{ref}, DE} \Delta P_{\text{loss}, DE}} \right]^{1/2}
\]  

(6)

Mass flow rate \( m_{\text{ref}} \) is found as follows:

\[
m_{\text{ref}} = \frac{\pi D_{\text{ref}, DE}^2 \rho_{\text{ref}, DE} \Delta P_{\text{loss}, DE}}{4}
\]  

(7)

However, in section AB, the mass flow rate is \( 3m_{\text{ref}} \), and

\[
u_{\text{ref}, AB} = \frac{\rho_{\text{ref}, D} \left[ D_{\text{ref}, DE}^2 \right]^{1/2}}{D_{\text{ref}, AB}} u_{\text{ref}, DE};
\]  

\[
\Delta P_{\text{loss}, AB} = 0.1582 \frac{\rho_{\text{ref}, D} \Delta P_{\text{loss}, DE} u_{\text{ref}, DE}^{1.75}}{D_{\text{ref}, AB}}
\]  

(8)

In section BC, the mass flow rate is \( m_{\text{ref}} \), and

\[
u_{\text{ref}, BC} = \frac{\rho_{\text{ref}, D} \left[ D_{\text{ref}, DE}^2 \right]^{1/2}}{D_{\text{ref}, BC}} u_{\text{ref}, DE};
\]  

\[
\Delta P_{\text{loss}, BC} = 0.1582 \frac{\rho_{\text{ref}, D} \Delta P_{\text{loss}, DE} u_{\text{ref}, DE}^{1.75}}{D_{\text{ref}, BC}}
\]  

(9)

and similar considerations apply for sections CD, EF, FG, GH, HI, and IJ.

Therefore, the total pressure at each point from A to J is as follows:

\[
P_{\text{tot, ref}, A} = P_{\text{ref}, A} + \frac{1}{2} \rho_{\text{ref}, A} u_{\text{ref}, AB}^2;
\]  

\[
P_{\text{tot, ref}, B} = P_{\text{tot, ref}, A} - \Delta P_{\text{loss}, AB}, \ldots, \text{ and}
\]

\[
P_{\text{tot, ref}, J} = P_{\text{tot, ref}, I} - \Delta P_{\text{loss}, IJ}
\]  

Then, the measured \( P_{\text{ref}, A} \) is known. \( P_{\text{ref}, B} \) and \( P_{\text{ref}, J} \) are found as follows:

\[
P_{\text{ref}, B} = P_{\text{ref}, B} + \frac{1}{2} \rho_{\text{ref}, B} u_{\text{ref}, AB}^2, \ldots \text{ and}
\]

\[
P_{\text{ref}, J} = P_{\text{ref}, J} + \frac{1}{2} \rho_{\text{ref}, J} u_{\text{ref}, BC}^2
\]  

At point C, temperature \( T_{\text{ref}, C} \) is measured and enthalpy \( H_{\text{ref}, C} \) is defined as follows:

\[
T_{\text{ref}, C} = T_C \text{ and } H_{\text{ref}, C} = H_{\text{ref}} \left( T_{\text{ref}, C} / P_{\text{ref}, C} \right)
\]  

(12)

At each point from A to E, each refrigerant tube is adiabatic. Therefore, the total enthalpy \( H_{\text{ref}, \text{ref}} \) at each point from A to E is the same. Thus, the enthalpy \( H_{\text{ref}} \) at each point from A to E is found as follows:

\[
H_{\text{ref}, A} + \frac{u_{\text{ref}, AB}^2}{2} = H_{\text{ref}, B} = H_{\text{ref}, C} + \frac{u_{\text{ref}, BC}^2}{2} = H_{\text{ref}, D} + \frac{u_{\text{ref}, CD}^2}{2} = H_{\text{ref}, E} + \frac{u_{\text{ref}, DE}^2}{2}
\]  

(13)

Therefore, temperature \( T_{\text{ref}} \) at each point from A to E can be calculated using \( H_{\text{ref}} \) and \( P_{\text{ref}} \). On the other hand, in each section from EF to IJ, heat inflows from the airfoil heat exchangers. The steady state basic equation in an integrated form for section EF, for example, is as follows:

\[
Q_{\text{convection, EF}} = Q_{\text{ref, EF}}
\]  

(14)

Based on the heat balance in section EF, the heat convection rate \( Q_{\text{convection, EF}} \) is

\[
Q_{\text{convection, EF}} = -m_{\text{ref}} \left[ H_{\text{tot, ref}} \left( T_{\text{ref}, E} / P_{\text{ref}, E} / u_{\text{ref}, DE} \right) - H_{\text{tot, ref}} \left( T_{\text{ref}, E} / P_{\text{ref}, E} / u_{\text{ref}, DE} \right) \right]
\]  

(15)
and the refrigerant heat gain rate $Q_{ref,EF}$ in section EF from the airfoil heat exchanger is

$$Q_{ref,EF} = \sum_{j} \{ Q_{ref,j} \} = \sum_{j} \left( \Delta h_{ref,j} \left[ T_{ref,EF} - T(j) \right] \right)$$

(16)

where $Q_{ref,j}$ was described in Eq. (4), and the summation of $Q_{ref,j}$ at all of the control volumes $j$ in contact with the refrigerant in section EF is used. Similar procedures can be applied for sections FG, GH, HI, and IJ.

**Adiabatic Air Temperature.** Although the airfoil heat exchanger surfaces are rigorously nonadiabatic, the air temperature in a boundary layer of a high-speed compressible airflow on a solid surface is close to the adiabatic air temperature. Recently, Pinilla et al. investigated the effects of the adiabatic temperature on the heat load of the blades of a gas turbine [21]. They mentioned that the adiabatic temperature plays a role in determining the heat flux through the air-contact surfaces.

On the outer air-contact surfaces, the adjacent local static air pressure can be calculated using the adjacent local pressure coefficient $S_j$ as follows:

$$P_{tot,air} = P_{tot,air} - \frac{1}{2} \rho_{air} \frac{V_{air}^2}{C_0}$$

(17)

where $S_j$ is obtained from Ref. [13] and interpolated. Note that a decrease in $S_j$ implies an adverse pressure gradient, and vice versa.

Thus, the adjacent local air Mach number and the adjacent local static air temperature are found as follows:

$$M_{air,j} = \sqrt{\frac{2}{\gamma - 1} \left( \frac{P_{tot,air,j}}{P_{tot,air}} - 1 \right) \frac{T_{tot,air}}{1 + \frac{\gamma}{2} M_{air,j}^2}}$$

(18)

The adjacent local adiabatic air temperatures $T_{air,adiabatic,j}$ in the air’s laminar or turbulent boundary layer is formulated as:

$$T_{air,adiabatic,j} = T_{air,j} + \left[ T_{tot,air} - T_{air,j} \right] r_j$$

(19)

where $r_j$ is the adjacent local recovery coefficient. Each $r_j$ represents a laminar or turbulent boundary layer as follows:

$$r_j = \frac{Pr_{1/2}}{Pr_{1/3}}$$

for air laminar boundary layer

$$r_j = \frac{Pr_{1/3}}{Pr_{1/3}}$$

for air turbulent boundary layer

(20)

If a pure air laminar flow enters the system, a transition to a turbulent boundary layer occurs across the maximum 5 point $X_{max,Lc}$, i.e., $Re_{air,transition} \approx X_{max,Lc}$ in a cascade of airfoils. In the present test setup, there may be many main flow instabilities. Thus, the transition may occur slightly upstream of $X_{max,Lc}$. However, it probably occurs not far from $X_{max,Lc}$, as shown in Fig. 6.

Here, $T_{air,adiabatic,j}$ was determined using only the airflow conditions, i.e., air Reynolds, Prandtl, and Mach numbers, and can be substituted into Eq. (5).

**Calculation Procedure of Inverse Heat Transfer Method.** The following procedure was conducted for the assumed $h_{air,j}$ and $h_{ref,j}$ to calculate the distribution of solid temperatures $T(j)$ in the airfoil heat exchanger, as well as to calculate the refrigerant temperatures $T_{ref}(F) - T_{ref}(J)$.

First, we calculated the already determined values according to the experimental conditions before using an inverse heat transfer method. The already determined values were the distribution of the adiabatic air temperatures $T_{air,adiabatic,j}$ around the airfoil heat exchanger, and the refrigerant temperatures $T_{ref}(F) - T_{ref}(J)$. The distribution of $T_{air,adiabatic,j}$ around the airfoil heat exchanger was given in the Adiabatic Air Temperature subsection when the air inlet conditions and cascade configuration were determined. $T_{ref,A} - T_{ref,E}$ were given in the Refrigerant Temperature subsection when the refrigerant inlet conditions were determined.

Second, for the assumed values of $h_{air,j}$ and $h_{ref,j}$, the solid temperatures $T(j)$ and the refrigerant temperatures $T_{ref}(F) - T_{ref}(J)$ were found numerically by solving Eqs. (1) and (14). Here, Eqs. (2), (4), and (5) were substituted into Eq. (1) at all of the solid control volumes in the airfoil heat exchanger, and Eqs. (15) and (16) were substituted into Eq. (14) at all of the refrigerant sections. Thus, we could generate simultaneous temperature equations for all the control volumes of the solid and refrigerant sections. To solve these, all of the $T$ coefficients were arranged for control volume $j$ as follows:

$$c_j T(j) + c_{p,j} T(p) + c_q T(q) + c_r T(r) + c_s T(s) = c_{jE} T_{ref,E} + c_{jT} T_{ref,T} + c_{jA} T_{air,adiabatic,j}$$

(21)

where only $T_{ref,E}$ and $T_{air,adiabatic,j}$ are known, and $T_{ref,T}$, i.e., $T_{ref}(F) - T_{ref}(J)$, are unknown. Therefore, a large coefficient matrix for all the airflow temperatures for all the control volumes of the solid and refrigerant sections was constructed. Then, this coefficient matrix was diagonalized. Finally, the distribution of the solid temperatures $T(j)$ in the airfoil heat exchanger and the refrigerant sections’ temperatures $T_{ref}(F) - T_{ref}(J)$ were obtained for the assumed $h_{ref,j}$ and $h_{air,j}$.

Third, the heat removal rate from the hot air $Q_{air,whole}$ and the input rate into the cold refrigerant $Q_{ref,whole}$ were found as follows:

$$Q_{air,whole} = \sum_j \left[ Q_{air,j} \right]$$

(22)

$$Q_{ref,whole} = m_{ref,DE} \left[ H_{ref} \left( T_{ref,1} - T_{ref,1} - T_{ref,1} \right) \right] - H_{ref,DE} \left( T_{ref,E} - T_{ref,E} - H_{ref,DE} \right)$$

(23)

**Finding Heat Transfer Coefficients by Least Square Method.** In the Calculation Procedure of Inverse Heat Transfer Method subsection, a procedure was described for using the assumed $h_{air,j}$ and $h_{ref,j}$ to calculate the solid temperatures $T(j)$, refrigerant sections’ temperatures $T_{ref}(F) - T_{ref}(J)$, and heat removal rate from the hot air $Q_{air,whole}$. In the present subsection, we discuss how to find the best combination of $h_{air,j}$ and $h_{ref,j}$. Here, the calculation results for $T(i)$, $T(ii)$, $T(iii)$, and $T(iv)$ are expressed in terms of the experimentally measured $T_{i1}$, $T_{i2}$, $T_{i3}$, and $T_{i4}$ values to facilitate the discussion. We divided the airfoil heat exchanger into three parts: the front part forward of tube IJ, the rear part behind tube EF, and the part between them, as shown in Fig. 6. Additionally, as shown in Fig. 6, the space-averaged heat transfer coefficients on the front part surface $h_{air,front}$, rear part surface $h_{air,rear}$, middle upper surface $h_{air,middle,up}$, and middle lower surface $h_{air,middle,low}$ were defined. $h_{ref}$ was locally determined based on...
the local adjacent refrigerant Reynolds number. However, the difference in the local refrigerant Reynolds numbers between the inlet and outlet was negligible. Thus, the space-averaged $\bar{h}_{ref}$ over all of the refrigerant-contact surfaces in the airfoil heat exchanger was considered.

In this study, the Levenberg–Marquardt algorithm [23] was used. It is one of the least square methods. In this algorithm, five fitting functions $f(1) = (T(i) - T_j)/T_i$, $f(2) = (T(i) - T_j)/T_i$, $f(3) = (T(iii) - T_{air})/T_{air}$, $f(4) = (T(iii) - T_j)/T_j$, $f(5) = \{Q_{air,whole} - Q_{ref,whole}\}/Q_{ref,whole}$ were set. The best combination of five independent variables $h_{air,front}$, $h_{air,middle,top}$, $h_{air,middle,low}$, $h_{air,rear}$, $h_{air}$ was found to realize the minimum of $f(1)^2 + f(2)^2 + f(3)^2 + f(4)^2 + f(5)^2$. In other words, we numerically found the calculation results that were the closest to the experimental results. This numerical analysis was performed by a VisualBasic2010 code that we developed using the Levenberg–Marquardt algorithm package provided by the ALGLIB Project [24].

Nusselt Numbers and Modified Stanton Numbers. Based on the calculation results, an average refrigerant Nusselt number $\bar{N}_{ref}$ and an average refrigerant modified Stanton number $\bar{S}_{ref}$ are obtained as follows:

$$
\bar{N}_{ref} = \frac{h_{ref} D_{ref}}{k_{ref}}, \quad \bar{S}_{ref} = \frac{h_{ref} \pi D_{ref} 5 W_{fringe}}{\rho_{ref} D_{ref} \eta C_{pref}} = \frac{\bar{N}_{ref} \pi D_{ref} W_{fringe}}{\rho_{ref} D_{ref} \bar{C}_{pref}} = \frac{20 W_{fringe}}{\bar{C}_{pref} \bar{D}_{ref}}
$$

(24)

where $\pi D_{ref} 5 W_{fringe}$ is the area of the refrigerant-contact surfaces. Here, $\bar{S}_{ref}$ is a dimensionless number that measures the ratio of the heat transferred to the refrigerant to the total heat capacity of the refrigerant passing through the airfoil heat exchanger.

The average $h_{air}$ over all the surfaces is calculated using the overall energy balance as follows:

$$
h_{air} = \frac{Q_{air,whole}}{\Delta T_{air}} = \frac{Q_{air,whole}}{\sum_{j} \Delta T_{air,j}}
$$

(25)

where $\Delta T_{air,j} = T_{air,adiabatic,j} - T(j)$ for control volume $j$ on the outer air-contact surfaces. Then, an average Nusselt number $\bar{N}_{air}$ and an average air modified Stanton number $\bar{S}_{air}$ are calculated as follows:

$$
\bar{N}_{air} = \frac{h_{air} L C}{k_{air}}, \quad \bar{S}_{air} = \frac{h_{air} L C W}{\rho_{air} L \sin(\beta) W_{air} C_{P_{air}}} = \frac{\bar{N}_{air} \pi D_{air} W_{air}}{\bar{C}_{air} \bar{D}_{air}}
$$

(26)

where $L C \sin(\beta) W$ is the cross section of the air stream handled by a unit of the airfoil heat exchanger. Here, the representative length of $R_{air,channel}$ is the height $L C \sin(\beta)$ of the air stream handled by a unit of the airfoil heat exchanger. $L_{air}$ is a dimensionless number that measures the ratio of the heat transferred to the air stream to the total heat capacity of the air stream handled by a unit of the airfoil heat exchanger.

Results and Discussion

Experimental Conditions. The carbon dioxide critical points were $T_{p,CO2} = 304.2$ K and $P_{p,CO2} = 7.38$ MPa, whereas the water critical points were $T_{p,water} = 647.3$ K and $P_{p,water} = 22.12$ MPa. The refrigerant temperature $T_{ref,E}$ and refrigerant pressure $P_{ref,E}$ were set to 315 K and 10, 20, and 30 MPa, respectively. They were chosen to verify the effects of the supercritical carbon dioxide and compressed water. The condition at point $E$ in the refrigerant tube corresponded to the inlet condition immediately upstream of the airfoil heat exchanger inlet, as shown in Fig. 4. Refrigerant Reynolds number $\Re_{ref,E}$ increased in proportion to the increasing refrigerant mass flow rate $m_{ref}$ for each refrigerant for each pressure because $\Re_{ref,E}$ depended only on the inlet velocity. Higher $\Re_{ref}$ values of $8.3 \times 10^4$ at 10 MPa, $4.8 \times 10^4$ at 20 MPa, and $3.8 \times 10^4$ at 30 MPa for carbon dioxide were achieved at the same $m_{ref}$ of 5 g/s compared with an $\Re_{ref}$ of $0.4 \times 10^4$ at all pressures for water. It is said that carbon dioxide has a better heat transfer performance in comparison with water because the Nusselt number is generally larger in a case with a larger Reynolds number. At 10 MPa, the specific heat $C_{P_{ref,E}}$ of 6096 J/kg·K for carbon dioxide was much larger than that of water (4156 J/kg·K). In contrast, the density $\rho_{ref}$ of 586.0 kg/m³ for carbon dioxide was much smaller than the value of 995.8 kg/m³ found for the water. The heat transport performance is proportional to $\rho_{ref} C_{P_{ref,E}}$, i.e., $\rho_{ref} C_{P_{ref,E}}$ for carbon dioxide and $\rho_{ref} C_{P_{ref,E}}$ for water, respectively. $\rho_{ref} C_{P_{ref,E}}$ for carbon dioxide was 4.14 at $10^6$ for the water. This produced a better heat transfer performance (through a heat transfer surface) for carbon dioxide compared with water, and almost the same heat transport performances (between different sites) at 10 MPa. Conversely, at 30 MPa, the $C_{P_{ref,E}}$ of 4113 J/kg·K for water was larger than the $C_{P_{ref,E}}$ of 4113 J/kg·K for carbon dioxide. Moreover, the density $\rho_{ref}$ of 902.7 kg/m³ for carbon dioxide approached the $\rho_{ref}$ of 1004 kg/m³ for water. The heat transport performance was $\rho_{ref} C_{P_{ref,E}} = 1.77 \times 10^6$ for carbon dioxide and $\rho_{ref} C_{P_{ref,E}} = 4.13 \times 10^6$ for water. Thus, carbon dioxide has a better heat transfer but less heat transport compared with water at 30 MPa. Given these results, we can say that carbon dioxide is superior to water as a heat transfer medium at all pressures, but the optimum heat transport medium should be determined on a case-by-case basis.

Figure 7 shows the air Prandtl number $\Pr_{air,in}$, Mach number $M_{air,in}$, and Reynolds number $R_{air,in}$ for different values of $\alpha$ within a range of 0°–25.5°. Here, the values of $\Pr_{air,in}$ were unrelated to the air temperature and pressure. Thus, the inlet air

![Fig. 7 Experimental Mach, Prandtl, and Reynolds numbers versus airflow direction angle from airfoil chord](image-url)
temperature could be maintained at a convenient value. Additionally, to conduct intercooling experiments, the adiabatic air temperature $T_{\text{air, adiabatic}}$ had to be higher than the refrigerant temperature $T_{\text{ref}}$. Therefore, the target inlet air total temperature $T_{\text{total, in}}$ was set to approximately 360 K. $M_{\text{air,in}}$ became smaller in regions far from $x = 10.5\,\text{deg}$. The airfoil incidence $\xi$ was 0 at $x = 9.47\,\text{deg}$. Thus, the air pressure loss in a closed wind tunnel increased in those regions far from $\xi = 0$. The same thing was true for $R_{\text{air,in}}$.

All of the experimental data were obtained after reaching dynamic steady states and thermal equilibrium. The following "experimental results" represent the results of inverse heat transfer analyses, in which the corresponding experimental results were used as boundary conditions and fitting parameters.

**Feature of Airfoil Heat Exchanger.** Figure 8 shows the temperature distributions for large and small values of $St_{\text{air}}$ under the same airflow conditions: $x = 10.5\,\text{deg}$, and $M_{\text{air,in}} = 0.60$. The left-hand figure in Fig. 9 shows the local air pressure coefficient $S$ distribution plotted from data taken from Ref. [13]. The others in Fig. 9 show the corresponding local air Mach number and adiabatic air temperature distributions on the air-contact surfaces plotted by using Eqs. (17)–(20) under the same conditions as those shown in Fig. 8. In this case, an air turbulent boundary layer formed over almost the entire lower surface. However, on the upper surface, the boundary layer temperature varied widely depending on the distance from each refrigerant tube along each heat flow path. Therefore, there were refrigerant tubes also became large. Then, the airfoil heat exchanger would be nearly the same as those when there is no refrigerant cooling. If, on the other hand, the refrigerant cooled the airfoil heat exchanger considerably, the temperature gradient (which is indicated by the distance from the black dotted line to the black solid and dashed lines in right-hand figure in Fig. 8) became large. The heat flow rate ($S_{\text{air}}$) between the middle surfaces and the refrigerant tubes also became small. Thus, the temperature profiles (which are the gray solid and dashed lines for $S_{\text{air}} = 0.00198$) remain smooth. $S_{\text{air}}$ would be nearly the same as those when there is no refrigerant cooling. If, on the other hand, the refrigerant cooled the airfoil heat exchanger considerably, the temperature gradient (which is indicated by the distance from the black dotted line to the black solid and dashed lines in right-hand figure in Fig. 8) became large. The heat flow rate ($S_{\text{air}}$) between the middle surfaces and the refrigerant tubes also became small. Then, the airfoil heat exchanger heat exchange considerably, the temperature gradient (which is indicated by the distance from the black dotted line to the black solid and dashed lines in right-hand figure in Fig. 8) became large. The heat flow rate ($S_{\text{air}}$) between the middle surfaces and the refrigerant tubes also became small. Then, the airfoil heat exchanger heat exchange considerably, the temperature gradient (which is indicated by the distance from the black dotted line to the black solid and dashed lines in right-hand figure in Fig. 8) became large. The heat flow rate ($S_{\text{air}}$) between the middle surfaces and the refrigerant tubes also became small. Then, the airfoil heat exchanger heat exchange considerably, the temperature gradient (which is indicated by the distance from the black dotted line to the black solid and dashed lines in right-hand figure in Fig. 8) became large. 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many temperature fluctuations on the middle surfaces in the airflow direction, such as the black solid and dashed lines for $S_{air} = 0.00348$. Thus, strong thermal entrance effects may occur in the air boundary layers on the middle surfaces, making $Nu_{air}$ larger, and the temperature gradient larger. Namely, there may be a positive feedback effect until the temperature on the middle surfaces approaches the air adiabatic temperature. This is why $Nu_{air,middle,low}$ and $Nu_{air,middle,up}$ increase with an increase in $St_{air}$ as shown in Fig. 10. This conclusion will be confirmed if transient experiments are performed in the future. However, the front and rear parts of the airfoil heat exchanger resemble forced-convection fins. Thus, a uniform temperature gradient forms, so that there are no temperature fluctuations along the airflow direction. Therefore, $Nu_{air,front}$ and $Nu_{air,rear}$ are determined depending on the airflow condition but independently of $St_{air}$.

Figure 11 shows the average air Nusselt number versus air Reynolds number at the outlet. The solid lines are from Ref. [12], summarizing the correlations of the air-cooled airfoil data obtained by Ainley [25], Fray and Barnes [26], Hodge [27], Wilson and Pope [28], Bammert and Hahnemann, and Andrews and Bradley [29], as well as data obtained by Turner himself. In addition, the dashed line is the correlation of the liquid-cooled airfoil data obtained by Freche and Diaguila [30]. Experiments were conducted in which the average airfoil temperature was held constant at 220$^\circ$F (378 K). The symbols in the figures indicate the results of our study, indicating a wide range of air Nusselt numbers even for the same air Reynolds number. In the air-cooled gas turbine, the cooling airflow rate was almost proportional to the main airflow rate. This was because the cooling air was bled from the compressor, such that the ratio of the airflow rates was almost constant.

The temperature effectiveness and airfoil temperature did not vary significantly. However, in the airfoil heat exchangers, the cooling refrigerant flow rate could be set independent of the main airflow rate. Furthermore, the refrigerant showed better a heat transfer performance than air. Lorenzini and Moretti conducted computer fluid dynamics calculations for fins in a liquid or air flow [14]. Their results seemed that the temperature distribution of the structure in a liquid varied over a wider range than that in air. The temperature distribution of the refrigerant-cooled airfoil heat exchanger varied over a wider range. Therefore, the air Nusselt numbers are affected not only by the air Reynolds and Prandtl numbers but also by the temperature fluctuations of the airfoil surfaces, as mentioned above. Freche and Diaguila also conducted other experiments while allowing the average airfoil temperature to vary widely by altering the refrigerant flow rate [30]. They found that the air Nusselt numbers were scattered even at the same air Reynolds number, although they suggested that experimental error caused this phenomenon. The reason for this scattering of the air Nusselt numbers found by Freche and Diaguila is likely to be the same as that found in our study. Basically, the air Nusselt number of a refrigerant-cooled airfoil heat exchanger cannot be determined using only the air Reynolds and Prandtl numbers.

Moreover, the right-hand figure in Fig. 11 shows the distribution of the average of all the $Nu_{air}$ values for each angle-of-attack $\alpha$ in the left-hand figure. The average in ranges far from $\alpha = 9.47$ deg corresponding to $\xi = 0$ may be higher than those in ranges close to $\alpha = 9.47$ deg. It was found that a larger absolute $\xi$ was preferred, unless a large separation occurred, from a heat transfer performance point of view.

**Design Method for Airfoil Heat Exchanger (Refrigerant Nusselt Number).** Figure 12 shows the refrigerant Nusselt number for a range of carbon dioxide refrigerant turbulent flows. It compares the refrigerant Nusselt numbers estimated according to Dittus–Boelter, Liao–Zhao, and our proposed correlations. The Liao–Zhao correlation [15] is

$$
Nu_{ref} = \frac{0.128Re_{ref}^{0.8}Pr_{ref}^{0.3}Gr_{ref}}{Re_{ref}^{0.25}Pr_{ref}^{0.437}}
$$

where the subscripts bulk and wall indicate values evaluated at bulk and wall temperatures, respectively. It covers carbon dioxide flow up to 12 MPa. In Fig. 12, the values calculated by the Dittus–Boelter correlation were underestimated. The thermal entrance regions through the airfoil heat exchanger in the refrigerant boundary layers and secondary Dean flows due to u-turn sections may have affected the enhancement of the refrigerant turbulent Nusselt number. On the other hand, the values calculated by the Liao–Zhao method were overestimated. Their method evaluates the buoyancy effect in a horizontal tube; however, our refrigerant tubes were set vertically. Therefore, our study’s buoyancy effect in the radial direction may be smaller than the Liao–Zhao estimation. Our correlation results for the carbon dioxide refrigerant Nusselt number were obtained as follows:

$$
Nu_{ref,turbulent} = 0.0231Re_{ref}^{0.823}Pr_{ref}^{0.300}
$$

by using a least square method for experimental carbon dioxide $Nu_{ref,turbulent}$. This correlation converged to within $-25\%$ and $+50\%$, relative to the experimental results. The average values of the current results were located between the Dittus–Boelter and Liao–Zhao correlations. The Dittus–Boelter correlation was acceptable, although there were slight differences in the constants compared with our correlation.

Our correlation results for the water refrigerant Nusselt number were obtained as follows:
by the same method as that used for carbon dioxide. Our correlation is within $-10\%$ and $+50\%$ compared with the experimental results. The values estimated by the Dittus–Boelter correlation were also underestimated for the previously given reason. However, the Dittus–Boelter correlation was acceptable, although there were also slight differences in the constants compared with our correlation.

**Design Method for Airfoil Heat Exchanger (Thermal Resistance).** The air Nusselt number $N_{u_{air}}$ for the refrigerant-cooled airfoil heat exchanger had a wide range, so it was difficult to predict $N_{u_{air}}$ by a correlation using only $Re_{air}$ and $Pr_{air}$. Instead, the relationships between the heat transfer rate $Q_{whole}$, a logarithmic mean temperature difference $\Delta T_{lm,whole}$, and the refrigerant Nusselt number $N_{ref}$ were used to estimate $N_{u_{air}}$.

Temperature changes in the air and refrigerant through the airfoil heat exchanger were evaluated by the temperature effectiveness $\phi_{air}$ and $\phi_{ref}$ as follows:

$$
\phi_{air} = \frac{T_{air,adiabatic,in} - T_{air,adiabatic,out}}{T_{air,adiabatic,in} - T_{ref,E}}, \quad \phi_{ref} = \frac{T_{ref,1} - T_{ref,E}}{T_{ref,adiabatic,in} - T_{ref,E}} ~ (30)
$$

Based on the heat balance of the airfoil heat exchanger, the relationships of $\phi_{air}$ and $\phi_{ref}$ are found as follows by using the ratio of the refrigerant heat capacity rate to air heat capacity rate $\epsilon_{RA}$:

$$
\phi_{air} = \epsilon_{RA} \phi_{ref}, \quad \epsilon_{RA} = \frac{m_{ref} C_{ref}}{m_{air} C_{air}} ~ (31)
$$

The heat input into the refrigerant $Q_{whole}$ is explained by using Eq. (30) as follows:

$$
Q_{whole} \approx m_{ref} C_{ref} [T_{ref,1} - T_{ref,E}] = m_{ref} C_{ref} \phi_{ref} [T_{air,adiabatic,in} - T_{ref,E}] ~ (32)
$$

The logarithmic mean temperature difference $\Delta T_{lm,whole}$ for a counter-flow heat exchanger was derived by its definition as follows:

$$
\Delta T_{lm,whole} = \Phi [T_{air,adiabatic,in} - T_{ref,E}] ~ (33)
$$

where the variable $\Phi$ is found as follows:

$$
\Phi = 1 \quad \text{for} \quad \epsilon_{RA} = 1, \quad \Phi = \frac{\phi_{ref} - \phi_{air}}{\ln \left( \frac{1 - \phi_{air}}{1 - \phi_{ref}} \right)} \quad \text{for} \quad \epsilon_{RA} \neq 1
$$

Here, based on its definition, the ideal overall heat transfer coefficient $\eta$ is

$$
\eta = \frac{1}{\epsilon_{RA} + \frac{1}{H_2O}} ~ (34)
$$

if the heat exchanger is an ideal counter-flow heat exchanger without thermal resistance in the airfoil heat exchanger’s solid material. Furthermore, the correction factor $\psi$ is the ratio of the practical heat transfer rate $Q_{whole}$ to the ideal heat transfer rate, which includes thermal resistance of the airfoil heat exchanger’s solid material, as follows:

$$
Q_{whole} = \psi A_{ref} \Delta T_{lm,whole} ~ (35)
$$

Generally, the overall heat transfer coefficient includes thermal resistance by using the term $\delta/k$ in its denominator, where $\delta$ is the thickness of the solid material, and $k$ is the thermal conductivity. However, it is difficult to define the value of $\delta$ because there is no constant thickness in the airfoil heat exchanger. Therefore, the correction factor $\psi$ is introduced instead of $\delta/k$. Equations (32) and (33) were substituted into Eq. (35). Thus, the practical overall heat transfer coefficient $\psi\eta$ is as follows:

$$
\psi \eta = \frac{m_{ref} C_{ref} \phi_{ref}}{A_{ref} \Phi} ~ (36)
$$

Therefore, from Eqs. (34) and (36), the air heat transfer coefficient is obtained by using the refrigerant heat transfer coefficient $h_{ref}$ derived from Eqs. (24), (28), and (29) as follows:

$$
h_{air} = A_{ref} A_{air} \frac{\phi_{ref} \Phi \Phi_{air}}{m_{ref} \delta / k} h_{ref} ~ (37)
$$

and air Nusselt number $N_{u_{air}}$ is calculated using Eq. (26).

The values of correction factor $\psi$ are obtained using a least square method for all experimental $\psi$ as follows:

$$
\psi = \frac{0.1236(0.02093)[\xi + 1]}{[\phi_{ref} - \exp(-0.5\epsilon_{RA})]} + 1 \quad \text{for} \quad 4.3 \times 10^3 \leq Re_{air} \leq 5 \times 10^5, Pr_{air} \approx 0.73 ~ (38)
$$
The heat transfer performance of two air-to-refrigerant airfoil heat exchangers and a recirculated heat transport refrigerant was experimentally investigated. This constituted a light and compact heat exchange system. The Reynolds and Mach numbers of the airflow were in the ranges of $4.3 \times 10^5 - 5 \times 10^5$ and $0.55 - 0.62$, respectively. Carbon dioxide or water was used as a refrigerant up to a pressure of 30 MPa. Thus, the carbon dioxide was supercritical, and the water was a compressed liquid over its critical pressure.

The heat transfer coefficients on the outer air-contact and inner refrigerant-contact surfaces were calculated using an inverse heat transfer method. The inverse heat transfer calculations were conducted under conditions that included our experimentally measured temperatures at four points and another researcher’s already reported pressure distribution around an airfoil. The correlation of the carbon dioxide Nusselt number on the inner refrigerant-contact surface was $N_{ref} = 0.0231 \times \frac{Re_{ref}^{0.36}}{Pr_{ref}}$, and that of the water Nusselt number was $N_{ref} = 0.0230 \times \frac{Re_{ref}^{0.36}}{Pr_{ref}}$. Their correlations were very close to the Dittus–Boelter correlation. Furthermore, we proposed a method to evaluate a correction factor corresponding to the thermal resistance of the air-to-refrigerant airfoil heat exchanger. The correction factor increased with an increase in the absolute incidence, which was the angle of the flow direction from the airfoil camber line at its leading edge. The correction factor increased and asymptotically approached a certain value close to unity with a decrease in the refrigerant temperature effectiveness. Conversely, the correction factor drastically approached zero with an increase in the refrigerant temperature effectiveness. This indicates the existence of a maximum permissible temperature effectiveness for each ratio of the refrigerant heat capacity rate to that of air. On the other hand, the correction factor increased with a decrease in the ratio of the refrigerant heat capacity rate to that of air, and vice versa.

These correlations and correction factor can be used for designing airfoil heat exchangers.

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**Nomenclature**

- $A$: area
- $c$: constant number
- $C_p$: isobaric specific heat
- $d$: distance between control volume centers
- $D$: refrigerant tube inner diameter
- $Gr$: Grashof number
- $h$: heat transfer coefficient
- $H$: enthalpy
- $k$: thermal conductivity
- $L$: refrigerant tube length
- $L_C$, $L_T$: airfoil chord length, tangential spacing between airfoils’ leading edges
- $m$: mass flow rate
- $M$: Mach number
- $Nu$: Nusselt number
- $P$: pressure
- $Pr$: Prandtl number
- $Q$: heat flow rate
- $r$: recovery coefficient
- $Re_{air}$: air Reynolds number whose representative length is airfoil chord length
- $Re_{air, channel}$: air Reynolds number whose representative length is air inlet height handled by the unit airfoil
- $Re_{air, nozzle}$: air Reynolds number whose representative length is nozzle hydraulic diameter
- $Re_{ref}$: refrigerant Reynolds number whose representative length is circular tube diameter

where $\xi$ is the incidence in degrees, which is the flow direction angle from the airfoil camber line at its leading edge. Under constant $Re_{air}$ and $Pr_{air}$, obviously, the correction factor $\psi$ increases with an increase in absolute incidence $\xi$ in Eq. (38). Figure 13 shows a profile example of $\psi$. Under constant $\xi$ and $Re_{air}$, $\psi$ increases and asymptotically approaches a certain large value as $\psi_{ref}$ decreases; conversely, $\psi$ drastically approaches zero as $\psi_{ref}$ increases. Please note that the value of $\psi$ estimated by Eq. (38) would be inappropriate at a value greater than $\psi_{ref}$ that results in $\psi = 0$. This indicates the existence of a maximum permissible $\psi_{ref}$ for each $Re_{air}$. On the other hand, under constant $\xi$ and $Pr_{air}$, $\psi$ increases as $\psi_{ref}$ decreases and vice versa. In Eq. (38), $\psi$ implicitly contains the effects of $Re_{air}$ and $Pr_{air}$. Basically, $\psi$ should explicitly include the $Re_{air}$ and $Pr_{air}$ terms. However, the effects cannot be expressed because experiments in small ranges of $Re_{air}$ and $Pr_{air}$ were conducted in this study.

To verify the adequacy of $\psi$, we can estimate $Nu_{air}$ from $\psi$, $Nu_{ref}$, and $Q_{whole}$ by using these relations. Figure 14 compares the experimental values of $Nu_{air}$ with values estimated using this procedure. These estimations are sufficiently accurate within $\pm 10\%$. When practically designing airfoil heat exchanger, conversely, $Q_{whole}$ can be estimated by using $Nu_{air}$, $\psi$, and $Nu_{air}$ if $Nu_{air}$ will be obtained by an appropriate method.
Subscripts

\( S \) = pressure coefficient, \( S = \left[ P_{\text{tot,air,in}} - P_{\text{air,local}} \right] / \left[ P_{\text{air,local}}^2 \right] \)

\( St \) = modified Stanton number

\( T \) = temperature

\( u \) = velocity

\( W, W_{\text{fringe}} \) = airfoil width, airfoil width with both side fringes

\( x, y, z \) = coordinates

\( X \) = position to airfoil chord length, leading edge is at \( X = 0 \), trailing edge is at \( X = 1 \)

Parentheses

\((j)\) = variable to be calculated at \( j \)th control volume of solid

\((n)\) = variable to be calculated at \( n \)th refrigerant section next to \( j \)th control volume (\( n \) is either \( E \) or \( I \))

\((T, P, u)\) = function of \( T, P, \) and \( u \)

Greek Symbols

\( \alpha \) = angle-of-attack in degrees, flow direction angle from airfoil chord

\( \beta\) = inlet angle in degrees, flow direction angle from perpendicular to airfoil row

\( \gamma \) = specific heat ratio

\( \Delta P_{\text{loss}} \) = pressure loss

\( \Delta T, \Delta T_{\text{lm}} \) = temperature difference, logarithmic mean temperature difference

\( \varepsilon_{\text{RA}} \) = ratio of refrigerant heat capacity flow rate to air heat capacity flow rate

\( \eta \) = ideal overall heat transfer coefficient

\( \theta \) = turning angle in degree

\( \mu \) = viscosity

\( \zeta \) = incidence in degrees, flow direction angle from airfoil camber line at its leading edge

\( \rho \) = density

\( \sigma \) = solidity \( L_c/L_d \)

\( \Sigma \) = summation

\( \phi_{\text{air}} \) = air temperature effectiveness,

\( \phi_{\text{air ref}} \) = refrigerant temperature effectiveness,

\( \phi \) = variable, \( \Phi = 1 \) for \( \varepsilon_{\text{RA}} = 1, \Phi = \left( \phi_{\text{air}} - \phi_{\text{ref}} \right) / \left( 1 - \phi_{\text{ref}} \right) \) for \( \varepsilon_{\text{RA}} \neq 1 \)

\( \psi \) = correction factor of practical overall heat transfer coefficient compared with \( \eta \)

References


